Initial Results on Fairness in Examination Timetabling

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1 Introduction

Examination timetabling problems have attracted many researchers during the last couple of decades, and especially since the work of Carter [2], and Carter, Laporte and Lee [3]. The problems are NP-complete and challenging, and so have nurtured different approaches and techniques; for a recent survey see [9].

In standard formulations the quality of a solution, from the student perspective, is given by an objective function which is a simple weighted sum of penalties for the timetable of each student. For example, in the classic Toronto benchmarks [3], the penalty per student is designed to reflect the natural desire of students that their exams do not take place too close together in time. Hence, minimising the objective is intended to maximise the average student satisfaction with the personal spread of examinations; however, it does not do anything to ensure equality between students. Students may consider the assessments to be unfair if some students have well spread out exams (small penalty) whilst others have many exams close together (large penalty). We believe that it is reasonable that overall student satisfaction could also be improved by increasing the fairness of treatment between students. In this paper, we make preliminary investigations of how to increase such fairness.

For general background, the common sense definitions of fairness in political science and political economics are discussed in [6] which defines fairness as an allocation where "no person in the economy prefers anyone else's consumption bundle over his own." In general resource allocation, there are two well-accepted and common notions of fairness criteria: *max-min* fairness and *proportional* fairness. *Max-min* fairness allows to say an allocation is fairer than another allocation but does not measure how much fairer [10], whilst *Proportional* fairness is quantitative measure of fairness (see [1] for details). Fairness has been studied before in combinatorial optimisation problems; for

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example, flight landing scheduling [12] and nurse rostering [4,11]. However, to the best of our knowledge, there have only been a few works dealing with fairness in educational timetabling. The most related prior work is for course timetabling by [8], and in which two formulations based on max-min fairness and the "Jain's Fairness Index" [7] are proposed. Here, rather than use max-min methods, in order to improve fairness in examination timetabling, we study the effects of modifying the objective function.

2 The Modified Objective Function

In order to take fairness into account, the basic idea in this preliminary investigation is by modifying the objective function. Instead of using standard linear summation (LS) objective function, a modified objective functions (MS) is studied.

$$LS = \sum_{s=1}^{S} P_s \tag{1}$$

$$MS = \sum_{s=1}^{S} f(P_s) \tag{2}$$

Where,

$$P_s = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} P(s,i,j) \tag{3}$$

$$P(s,i,j)) = \begin{cases} W_{|t_j - t_i|} & \text{if student s takes exams i and j} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Here, S is the total number of students, P_s is total penalty of the s'th student, N is the number of examinations, $P(s,i,j)$ is the penalty of the s'th student taking examination i and j, and t_i is a timeslot where an examination i is scheduled. $W_{|t_j-t_i|}$ is the weight whenever a student sits two examinations that are scheduled $\mid t_j - t_i \mid$ apart. The penalty weight, $W_{|t_j-t_i|}$ is calculated as $2^{5-|t_j-t_i|}$ where, $\mid t_j - t_i \mid \in \{1,2,3,4,5\}$. $W_{|t_j-t_i|}$ is equal to 0 if $\mid t_j - t_i \mid > 0$..

The function $f(P_s)$ is the function of total individual student penalty and that can be used to encourage fairness by using a non-linear function that increases the penalty on larger initial penalties. In this study, we considered the fairly standard case of $f(P_s)$ as Sum of Squares (SoS), but also the more general Sum of Powers (SoP):

$$\text{SoS}: \quad f(P_s) = P_s^2 \tag{5}$$

$$\text{SoP(p)}: \quad f(P_s) = P_s^p \tag{6}$$

3 Experimental Methods and Results

The examination timetabling solver used for this paper is a standard two phase algorithm, consisting of a sequential heuristic construction method followed by an improvement phase based on the great deluge algorithm [5]. We use Jain's Fairness Index (JFI) [7] to measure the fairness of the solutions. Suppose a solution from a timetabling solver is T, the JFI of the solution is defined as follows in terms of the penalties, P_s, for the individual students:

$$JFI(T) = \frac{\left(\sum_{s=1}^{S} P_s\right)^2}{\left(S * \sum_{s=1}^{S} (P_s)^2\right)} \tag{7}$$

Note that the JFI value is bounded between 0 and 1, with larger values implying the solution becomes more fair. A value of 1 corresponds to complete fairness.

Table 1 Experimental result: solutions' fairness produced from LS, SoS, and SoP objective function

Inst.	LS		SoS		SoP	
	MEAN	JFI	MEAN	JFI	MEAN	JFI
CAR91	5.46	0.33	5.56	0.35	8.33	0.39
CAR92	4.96	0.29	4.83	0.31	6.74	0.35
EAR83	39.8	0.82	39.42	0.84	47.88	0.86
HEC92	11.24	0.49	11.34	0.52	15.32	0.58
KFU93	16.25	0.54	16.26	0.56	20.07	0.63
LSE91	13.36	0.53	13.42	0.57	17.16	0.63
PUR93	5.84	0.35	6.06	0.38	8.62	0.44
RYE92	9.82	0.37	9.7	0.39	16.21	0.44
STA83	163.08	0.91	163.12	0.91	168.01	0.93
TRE92	8.82	0.44	8.93	0.47	11.16	0.49
UTA92	4.23	0.24	4.31	0.26	5.22	0.29
UTE92	26.54	0.79	26.51	0.8	31.59	0.81
YOR83	39.12	0.75	38.8	0.77	48	0.77

In this preliminary study, the experiment is carried out under Carter dataset [3]. In the experiments, for each instance in the dataset we conduct 20 test runs. The results are given in Table 1, and compares the mean penalty cost (MEAN) and Jain's Fairness Index (JFI) of solutions resulting from three different objective functions, specifically, linear summation (LS), Sum of Squares (SoS) and Sum of Powers (SoP).

It can be seen from Table 1 that both SoS and SoP objective function can produce fairer solutions for all problem instances, though sometimes with some loss of quality of the standard linear sum. Not surprisingly, SoP can produce fairer solution than SoS, however, it was somewhat surprising that we had to go as high as the 16th power to get a significant effect, and that the common SoS approach was not always sufficient to improve the fairness significantly. However, SoS compensates with a smaller increase in the average linear penalty cost than with SoP, and even produced smaller average such as in instance UTE92, TRE92, and EAR83 – though this may just be an artefact of the search algorithm used.

4 Conclusions

This paper has presented a preliminary study which attempts to improve fairness in examination timetabling solutions. A modified objective function is proposed instead of traditional linear summation of penalties, and the experimental results show this approach produced fairer solution for almost all problem instances. However, not unexpectedly, it also shows that fairer solution can also cause a slightly worse overall average quality of solution. Naturally, the problem is one of multi-objective with a balance to be struck between quality and fairness. This work encourages future investigation of search methods for finding fairer solutions. In future studies, in addition to the student perspective, it may also important to take into consideration the quality and fairness of solution from the perspective of other stakeholders such as invigilators, lecturers, and estates, and also in more realistic formulations [**?**].

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